



(Formal) Software Verification via Logic

(using One-Counter Automata)

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1 Formal Verification

2 One-Counter Automata

3 Logic

4 Conclusion

1. Formal Verification

Motivation

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3 def dummy(x:int,y:int):
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“Program testing can be a very effective way to show the presence of bugs, but it is hopelessly inadequate for showing their absence.” - E. Dijkstra

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- A lot more at https://en.wikipedia.org/wiki/List_of_software_bugs

Formal Verification



Physics Mathematics Biology Computer Science Topics Archive

COMPUTER SECURITY

Hacker-Proof Code Confirmed

55 |

Computer scientists can prove certain programs to be error-free with the same certainty that mathematicians prove theorems. The advances are being used to secure everything from unmanned drones to the internet.



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Formal verification is the process of proving the correctness of intended algorithms underlying a system with respect to a certain formal specification or property, using formal methods of mathematics.

Formal Verification

Formal Models

- Finite State Machines
- Vector Addition Systems
- Timed Automata/ Hybrid Automata
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Requirements/Formal Specifications

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Advantages of Formal Verification

- Formally proving correctness and ensure safety
- Significantly reduces the verification time

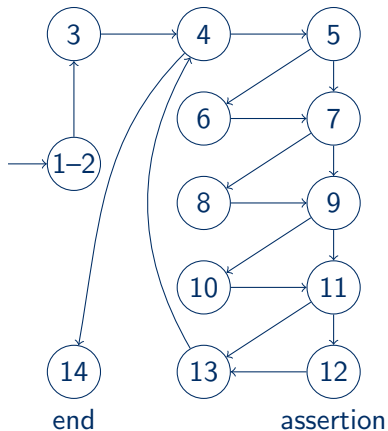
2. One-Counter Automata

Using the control flow graph (CFG)

```
1 skip = 2
2 retake = 3
3 retake += skip
4 while retake >= 0:
5     if retake == 3:
6         print("You get a reminder")
7     if retake == 4:
8         print("You get soft warning")
9     if retake == 5:
10        print("You get hard warning")
11    if retake >= 6:
12        assert("God forbid!")
13    retake -= 1
14 # end program
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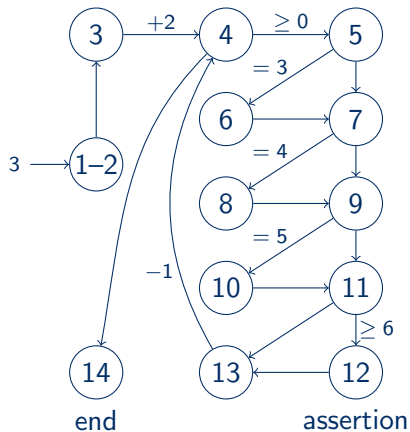

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Extending the CFG with a counter

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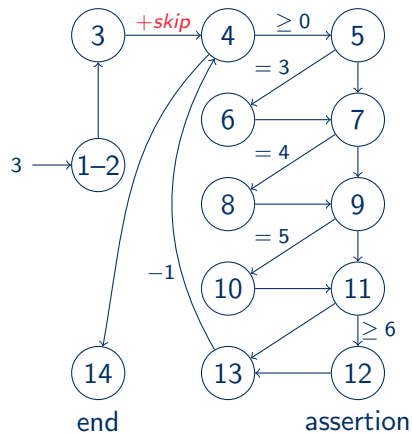
Counter : = retake

Parametric one-counter automata

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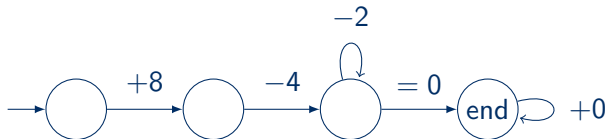
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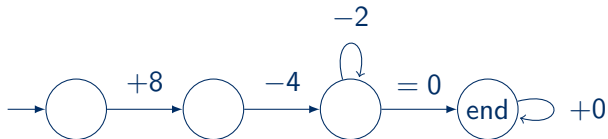


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(Parametric) One-Counter Automata



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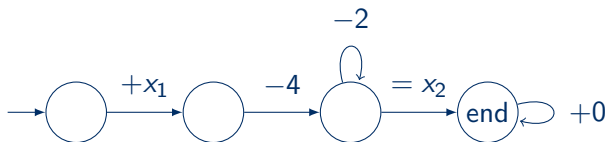


Counter Value has to be non-negative all the time!

(Parametric) One-Counter Automata

Natural-valued parameters

$$X = \{x_1, \dots, x_n\}$$



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Decidability Questions

Definition (Parameter-value Reachability)

Is there some valuation $V : X \rightarrow \mathbb{N}$ such that there is some run of \mathcal{A} that reaches/avoids a good/bad state?

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Non-parametric Versions of the above also

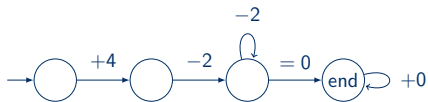
3. Logic

Towards Logic

“Have a problem? Encode it into a logic with decidable theory.”

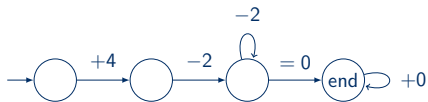
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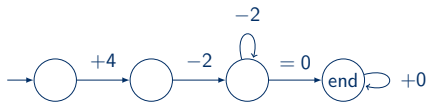


Logical formula:

$$\exists k(4 - 2 - 2k = 0)$$

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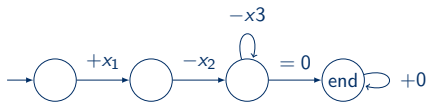
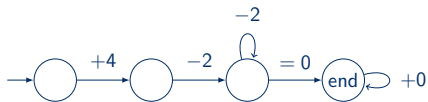
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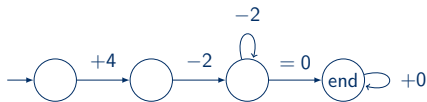
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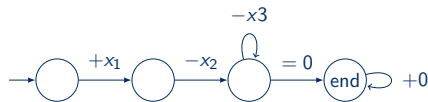
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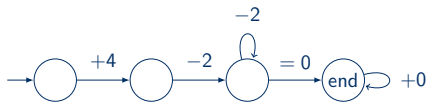
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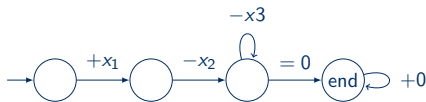
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Presburger Arithmetic with divisibility:

$$\text{PA} + \left| \begin{array}{l} \\ (a | b \iff \exists c \in \mathbb{Z} : b = ac) \end{array} \right.$$

Complexity and Decidability

- Non-parametric Reachability: **NP** (Presburger Arithmetic/PA)
- Non-parametric Synthesis: **coNP** (Reduction complement to Non-parametric Reach)
- Parametric Reachability: **NEXP** (Existential PAD)
- Parametric Synthesis: **N2EXP** (BIL : a fragment of one-alternation PAD)

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Research:

- Continuous One-counter automata, VASS
- Markov Decision Process
- Hybrid Automata